

D7- $\overline{D7}$ bilayer: holographic dynamical symmetry breaking

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We consider a holographic model of dynamical symmetry breaking in 2+1-dimensions, where a parallel D7-anti-D7 brane pair fuse into a single object, corresponding to the $U(1) \times U(1) \rightarrow U(1)$ symmetry breaking pattern. We show that the current-current correlation functions can be computed analytically and exhibit the low momentum structure that is expected when global symmetries are spontaneously broken. We also find that these correlation functions have poles attributable to infinite towers of vector mesons with equally spaced masses.

In weakly coupled quantum field theory, spontaneous symmetry breaking is a familiar paradigm. It is based on formation of a condensate, usually an order parameter obtaining a nonzero expectation value and the resulting features of the spectrum such as goldstone bosons and a Higgs field. String theory holography has given an alternative picture of dynamical symmetry breaking in terms of geometry. Particularly with probe branes, the symmetry breaking corresponds to the branes favoring a less symmetric worldvolume geometry over a more symmetric one. This is seen in the Sakai-Sugimoto model of holographic quantum chromodynamics [1]. There, chiral symmetry breaking corresponds to the fact that a $D8-\overline{D8}$ brane pair prefer to fuse into a cigar-like geometry, rather than remaining in a more symmetric independent configuration. In this paper, we shall study a model which is close in spirit to the Sakai-Sugimoto model, the $D7-\overline{D7}$ system which has a 2+1-dimensional overlap with a stack of $D3$ -branes. It can be considered a toy model of chiral symmetry breaking in strongly coupled 2+1-dimensional quantum field theories containing fermions and it is explicitly solvable. The symmetry breaking pattern is $U(N) \times U(N) \rightarrow U(N)$ and, at least in principle, it is possible to gauge various subgroups of the global symmetry group and to study the Higgs mechanism at strong coupling. In the following we shall concentrate on the case $U(1) \times U(1) \rightarrow U(1)$ which displays the essential features of the mechanism.

Before analyzing the $D7-\overline{D7}$ system, let us discuss its quantum field theory dual, the bilayer system depicted in figure 1. Massless relativistic 2+1-dimensional fermions are confined to each of two parallel but spatially separated layers. They are two-component spinor representations of the $SO(2,1)$ Lorentz group with a $U(1)$ global symmetry for the fermions inhabiting each layer. The overall global symmetry is thus $U(1) \times U(1)$. The 3+1-dimensional bulk contains $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. The fermions transform in the fundamental representation of the gauge groups of the Yang-Mills theories. As shown in figure 1, the rank of the Yang-Mills gauge groups differ in the interior and exterior of the bilayer by an integer k which arises from the worldvolume flux in the $D7-\overline{D7}$ system. The D-brane system which we shall discuss studies this theory in the strong coupling

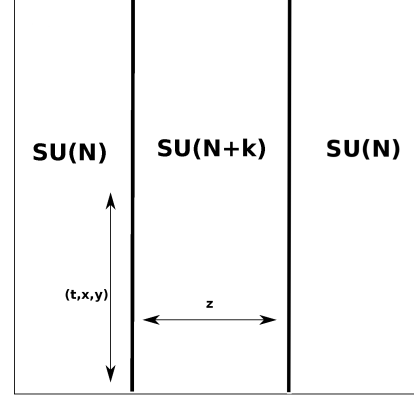


FIG. 1: Two parallel 2-dimensional spaces, depicted by the vertical dark lines, are inhabited by fundamental representation fermions which interact via fields of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in the bulk. The Yang-Mills theory in the region between the layers has a different rank gauge group than that in the regions external to the bilayer.

planar limit where, first, the Yang-Mills coupling g_{YM} is taken to zero and N to infinity while holding $\lambda \equiv g_{YM}^2 N$ fixed and, subsequently, a strong coupling limit of large λ is taken. The field theory mechanism for the symmetry breaking which we shall analyze is an exciton condensate which binds a fermion on one layer to an anti-fermion on the other layer and breaks the $U(1) \times U(1)$ symmetry to a diagonal $U(1)$.

There has been significant recent interest in graphene bilayer systems where formation of an exciton driven dynamical symmetry breaking of the kind that we are discussing has been conjectured [2]. The geometry is similar, with the layers in figure 1 replaced by graphene sheets and the space in between with a dielectric insulator. In spite of some differences: graphene is a relativistic electron gas with a strong non-relativistic Coulomb interaction, whereas what we describe is an entirely relativistic non-Abelian gauge theory, there are also similarities and perhaps lessons to be learned. For example, we find that the exciton condensate forms in the strong coupling limit even in the absence of fermion density whereas the weak coupling computations that analyze graphene need nonzero electron and hole densities in the sheets to create

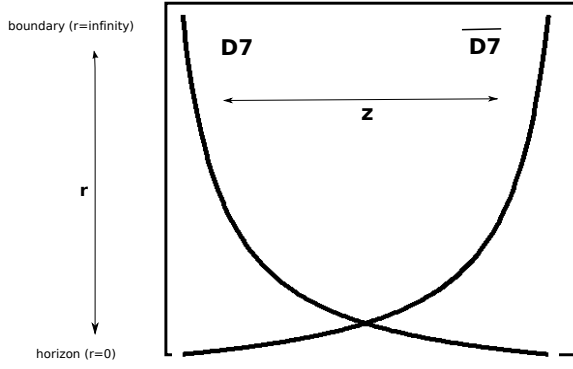


FIG. 2: The z -position of the D7-branes depends on AdS-radius and with the appropriate orientation the branes would always intersect.

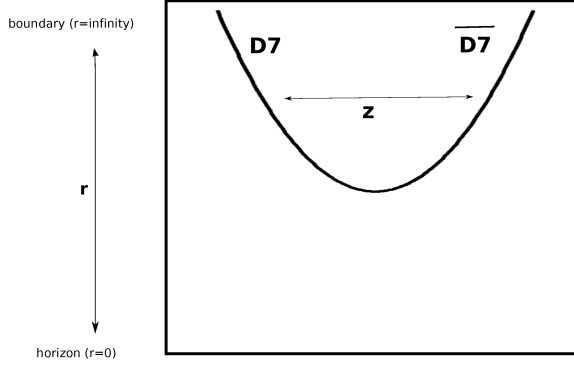


FIG. 3: Joined configuration.

an instability. We also find “coulomb drag”, where the existence of an electric current in one layer induces a current in the other[3]. In the holographic model, the drag would vanish in the absence of a condensate, whereas it is large when a condensate is present. The correlator between the electric currents in the two sheets (from (10) below) is

$$\langle j_a(k) \tilde{j}_b(\ell) \rangle = \frac{4\lambda(1+f^2)|k|}{(2\pi)^2 \sinh 2|k|\rho_m} \left(\delta_{ab} - \frac{k_a k_b}{k^2} \right) \delta(k+\ell) \quad (1)$$

where $|k| = \sqrt{\vec{k}^2 - \omega^2/v_F^2}$, there is a factor of 4 from the degeneracy of graphene, v_F is the electron fermi velocity and λ and f^2 are parameters and ρ_m , given in (9), is proportional to the interlayer spacing. Aside from the superfluid pole at $k^2 = 0$, this correlator has an infinite series of poles at $k^2 = (n\pi/\rho_m)^2$, $n = 1, 2, \dots$ due to vector mesons. Parameters partially cancel in the ratio of the current-current correlator in (1) to the single layer correlator, $\langle \tilde{j}\tilde{j} \rangle / \langle jj \rangle = \text{csch} 2|k|\rho_m$.

Symmetry breaking in the $D7$ - $\overline{D7}$ system has already been studied in reference [4]. The mechanism is a joining of the $D7$ and $\overline{D7}$ worldvolumes as depicted in figure 3. The $D7$ and $\overline{D7}$ are probe branes [5] in the $AdS_5 \times S^5$ geometry which is the holographic dual of 3+1-dimensional

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory. A single probe D7-brane is stable when it has magnetic flux added to its worldvolume [6]. Its most symmetric configuration is dual to a defect conformal field theory [6][7] where the flux (f in the following) is an important parameter which determines, for example, the conformal dimension of the fermion mass operator. The $D7$ - $\overline{D7}$ pair would tend to annihilate and are prevented from doing so by boundary conditions that contain a pressure (the parameter P in the following) which holds them apart. The problem to be solved is that of finding the configuration of the $D7$ and $\overline{D7}$ in the $AdS_5 \times S^5$ background, subject to the appropriate boundary conditions. We shall impose the parity and time-reversal invariant boundary conditions that were discussed in reference [7]. We differ from reference [4] in that we use the zero temperature limit, a simplification that allows us to obtain our main result, explicit current-current correlation functions for the theory described by the joined solution (11)-(13). The $AdS_5 \times S^5$ metric is

$$ds^2 = R^2[r^2(-dt^2 + dx^2 + dy^2 + dz^2) + \frac{dr^2}{r^2} + d\psi^2 + \sin^2 \psi d\Omega_2^2 + \cos^2 \psi d\tilde{\Omega}_2^2] \quad (2)$$

where $d\Omega_2^2$ and $d\tilde{\Omega}_2^2$ are metrics of unit 2-spheres and $\psi \in [0, \frac{\pi}{2}]$. The radius of curvature is $R^2 = \sqrt{4\pi g_s N \alpha'}$, where g_s is the closed string coupling constant and N the number of units of Ramond-Ramond 4-form flux of the IIB string background. The holographic dictionary sets $g_{YM}^2 = 4\pi g_s$, and N becomes the rank of the Yang-Mills gauge group. The embedding of the $D7$ in this space is mostly determined by symmetry. We take the $D7$ and $\overline{D7}$ embeddings to wrap (t, x, y) , S^2 and \tilde{S}^2 and to sit at the parity symmetric point $\psi = \frac{\pi}{4}$. To solve embedding equations, the transverse coordinate z must depend on the radius r . At the boundary of AdS_5 ($r \rightarrow \infty$), we impose the boundary condition that the $\overline{D7}$ is located at $z = -L/2$ and $D7$ at $z = L/2$. The worldvolume metric of one of the branes is then

$$d\sigma^2 = R^2[r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}(1 + r^4 \dot{z}(r)^2) + \frac{1}{2}d\Omega_2^2 + \frac{1}{2}d\tilde{\Omega}_2^2] \quad (3)$$

where $\dot{z} = dz/dr$. The field strength of the world-volume gauge fields are

$$F = \frac{R^2}{2\pi\alpha'} \frac{f}{2} \Omega_2 + \frac{R^2}{2\pi\alpha'} \frac{f}{2} \tilde{\Omega}_2 \quad (4)$$

where Ω_2 and $\tilde{\Omega}_2$ are the volume forms of the unit 2-spheres. The flux forms two Dirac monopole bundles, each with monopole number $n_D = \sqrt{\lambda} f^2$. Stability and other properties of the theory [6][7] require that $23/50 \leq f^2 \leq 1$, otherwise it is a tunable parameter. The embedding is determined by extremizing the Dirac-

Born-Infeld plus Wess-Zumino actions,

$$S = -\frac{T_7 N}{g_s} \int d^8 \sigma \left[\sqrt{-\det(g(\sigma) + 2\pi\alpha' F)} \right. \\ \left. \mp \frac{(2\pi\alpha')^2}{2} F \wedge F \wedge C_4 \right] \quad (5)$$

$T_7 = 1/(2\pi)^7 \alpha'^4$ is the brane tension, C_4 is the Ramond-Ramond 4-form of the IIB string background and the \mp refer to the $D7$ and $\overline{D7}$, respectively. With our Ansatz, this reduces to a variational problem with Lagrangian

$$\mathcal{L} = (1 + f^2)r^2 \sqrt{1 + r^4 \dot{z}(r)^2} \mp f^2 r^4 \dot{z}(r) \quad (6)$$

$z(r)$ is a cyclic variable whose equation of motion is solved by $z_{\pm}(r) = \pm \frac{L}{2} \mp \int_r^\infty dr \dot{z}_{\pm}(r)$ is the position of the brane to the right (upper sign) or left (lower sign) of $z = 0$ and $\dot{z}_{\pm}(r) = \pm \frac{f^2 r^4 + P}{r^2 \sqrt{(r^4 - P)((1 + 2f^2)r^4 + P)}}$. P is an integration constant proportional to the pressure needed to hold the branes with their asymptotic separation L . When they are not joined, they do not interact, at least in this classical limit, and P must be zero. Then $z_{\pm}(r) = \pm \frac{L}{2} \mp \frac{f^2}{\sqrt{1 + 2f^2} r}$ as depicted in figure 2. When they are joined, as depicted in figure 3, P must be nonzero and they are joined at a minimum radius $r_0 = P^{\frac{1}{4}}$ and L and P are related by $LP^{\frac{1}{4}} = 2 \int_1^\infty dr \frac{f^2 r^4 + 1}{r^2 \sqrt{(r^4 - 1)((1 + 2f^2)r^4 + 1)}}$.

The joined solution will always be the lower energy solution when the branes are oriented as in figures 2 and 3. They are also stable for any value of L when the brane and antibrane are interchanged, the “chubby solutions” discussed in reference [4], only when $23/50 \leq f^2 \lesssim .56$. When $f^2 > .56$ the chubby solutions are unstable for any L . (As noted in reference [4], there can be a much richer phase structure when temperature, density or external magnetic fields are introduced.) For the chubby solution, the gauge group ranks N and $N + k$ in figure 1 trade positions.

A simple diagnostic of the properties of the fermion system in the strongly coupled quantum field theory which is dual to the joined branes is the current-current correlation function. It is obtained by solving the classical dynamics of the gauge field on the world-volume of the branes with the Dirichlet boundary condition. The quadratic form in boundary data in the on-shell action yields the current-current correlator. Here, the brane geometry is simple enough that, to quadratic order, AdS components of the vector field decouple from the fluctuations of the worldvolume geometry, as well as from those components on S^2, \tilde{S}^2 . To find them, we simply need to solve Maxwell’s equations on the worldvolume,

$$\partial_B [\sqrt{g} g^{BC} g^{DE} (\partial_C A_E - \partial_E A_C)] = 0$$

where the worldvolume metric is given in equation (3) above and the gauge fields have indices $B, C, \dots =$

(t, x, y, r) . In the $A_r = 0$ gauge,

$$\partial_r (\partial_a A_a) = 0 \quad , \quad \partial_\rho^2 A_a + \partial_b (\partial_b A_a - \partial_a A_b) = 0 \quad (7)$$

where indices $a, b, \dots = (t, x, y)$, we have suppressed the Minkowski metric for contracted indices and we have redefined the radial coordinate as $\rho = \int_r^\infty \frac{dr}{r^2} \sqrt{1 + r^4 \dot{z}^2}$. In the simpler case of a single D7-brane, say the brane which originates on the right in figure 2, whose geometry is AdS_4 , these equations are solved by [7]

$$A_a(k, \rho) = A_a(k) \cosh |k| \rho + \frac{1}{|k|} A'_a(k) \sinh |k| \rho$$

where $A_a(k, \rho) = \int d^3 x e^{ikx} A_a(x, \rho)$, $k_a A_a(k) = 0 = k_a A'_a(k)$ and $|k| = \sqrt{\vec{k}^2 - k_0^2}$. Regularity at the Poincare horizon ($\rho \rightarrow \infty$) requires $A'_a(k) = -|k| A_a(k)$. Moreover, with the on-shell action,

$$S = -\frac{N(f^2 + 1)}{4\pi^2} \int d^3 k |k| A_a(-k) (\delta_{ab} - k_a k_b / k^2) A_b(k) + \dots$$

e^{-S} is a generating function for current-current correlators in the dual conformal field theory where the $U(1)$ symmetry is global, ($j_a(k) = g_{YM} \delta / \delta A_a(-k)$)

$$\langle j_a(k) j_b(\ell) \rangle = \frac{\lambda(f^2 + 1)}{2\pi^2} |k| (\delta_{ab} - k_a k_b / k^2) \delta(k + \ell) \quad (8)$$

Alternatively, if instead of the Dirichlet boundary conditions used above, we impose the Neuman boundary condition that $\partial_\rho A_a(k, \rho)$ approaches $A'_a(k)$ as $\rho \rightarrow 0$, we can write the on-shell action as a functional of $A'(k)$ and it generates correlators of the gauge field in a different conformal field theory where the $U(1)$ symmetry is gauged and the gauge field is dynamical. It yields the Landau gauge 2-point function of the photon field in that theory [9] ($a_a(k) = \delta / \delta A'(-k)$),

$$\langle a_a(k) a_b(\ell) \rangle = \frac{N(f^2 + 1)}{2\pi^2} \frac{1}{|k|} (\delta_{ab} - k_a k_b / k^2) \delta(k + \ell)$$

The momentum dependence of these correlation functions is consistent with conformal symmetry.

To analyze the joined configuration, we note that in that case ρ reaches a maximum

$$\rho_m = \frac{L}{2} \frac{\int_0^1 \frac{dx(1+f^2)}{\sqrt{(1-x^4)((1+2f^2)-x^4)}}}{\int_0^1 \frac{dx(f^2+x^4)}{\sqrt{(1-x^4)((1+2f^2)-x^4)}}} \quad (9)$$

We use a variable $s = \rho$ for the left branch and $s = 2\rho_m - \rho$ for the right branch of figure 3. With the Dirichlet boundary conditions $A_a(k, s = 0) = A_a(k)$ and $A_a(k, s = 2\rho_m) = \tilde{A}_a(k)$ the on-shell action is

$$\tilde{S} = -\frac{N(f^2 + 1)}{4\pi^2} \int d^3 k \left[(|A_a(k)|^2 + |\tilde{A}_a(k)|^2) \coth 2|k| \rho_m \right. \\ \left. - 2A_a(-k) \tilde{A}_a(k) \operatorname{csch} 2|k| \rho_m \right] + \dots \quad (10)$$

The current-current correlation functions can be diagonalized by $j_+ \equiv j + \tilde{j}$, $j_- \equiv j - \tilde{j}$, so that

$$\langle j_{a+} j_{b-} \rangle = 0 \quad (11)$$

$$\langle j_{a+} j_{b+} \rangle = \frac{\lambda(f^2 + 1)}{2\pi^2} k \tanh k\rho_m \left(\delta_{ab} - \frac{k_a k_b}{k^2} \right) \quad (12)$$

$$\langle j_{a-} j_{b-} \rangle = \frac{\lambda(f^2 + 1)}{2\pi^2} k \coth k\rho_m \left(\delta_{ab} - \frac{k_a k_b}{k^2} \right) \quad (13)$$

At large Euclidean momenta, (12) and (13) revert to the conformal field theory correlators in (8). At time-like momenta the correlator $\langle j_{a-} j_{b-} \rangle$ has a pole at $k^2 = 0$ which is the signature of dynamical breaking of a diagonal $U(1)$ subgroup of the $U(1) \times U(1)$ symmetry and gives rise to superfluid linear response. On the other hand, the correlator $\langle j_{a+} j_{b+} \rangle \sim k^2$ for small k , which indicates that the system is an insulator in the channel which couples to the other diagonal $U(1)$ subgroup with current j_{a+} . In addition, both correlators have an interesting analytic structure. They have no cut singularities. $\langle j_{a+} j_{b+} \rangle$ has poles at the energies

$$k_0^2 = k_1^2 + k_2^2 + \left(\frac{\pi(2n+1)}{2\rho_m} \right)^2, \quad n = 0, 1, \dots \quad (14)$$

and $\langle j_{a-} j_{b-} \rangle$ has poles at

$$k_0^2 = k_1^2 + k_2^2 + \left(\frac{\pi n}{\rho_m} \right)^2, \quad n = 0, 1, \dots \quad (15)$$

indicating two infinite towers of massive spin-one particles. These would be narrow bound state resonances with decay widths that vanish as $N \rightarrow \infty$, as one expects in the large- N limit that we are studying here [8]. The current operators create these single-particle states from the vacuum. Their creation of multi-particle states, which would normally result in cut singularities, is suppressed in the large N planar limit. The resonances are

simply the tower of vector mesons whose masses (14) and (15) occur at eigenvalues of $-\partial_s^2$ with Dirichlet boundary conditions on the interval $s \in [0, 2\rho_m]$. The fact that currents create either even or odd harmonics is due to $L \rightarrow -L$ reflection symmetry.

In the above, we used Dirichlet boundary conditions for the worldvolume gauge field. It is possible, alternatively, to select Neumann boundary conditions by choosing $\partial_s A_a$ rather than A_a on the asymptotic boundary. The result is dual to a field theory where the $U(1)$ symmetries are gauged and the on-shell action generates photon correlation functions [9]. Most relevant are mixed Neuman and Dirichlet boundary conditions. For example, in graphene, a diagonal electromagnetic $U(1)$ is gauged whereas the orthogonal $U(1)$ is a global symmetry. This is obtained by applying the Dirichlet condition to $A(s=0, k) - A(s=2\rho_m, k)$ and the Neuman condition to $\partial_s A(s=0, k) - \partial_s A(s=2\rho_m, k)$. In this case, the correlation functions are

$$\langle j_a a_b \rangle = 0 \quad (16)$$

$$\langle j_a j_b \rangle = \frac{\lambda(f^2 + 1)}{4\pi^2} k \coth k\rho_m \left(\delta_{ab} - \frac{k_a k_b}{k^2} \right) \quad (17)$$

$$\langle a_a a_b \rangle = \frac{N(f^2 + 1)}{4\pi^2} \frac{1}{k} \coth k\rho_m \left(\delta_{ab} - \frac{k_a k_b}{k^2} \right) \quad (18)$$

The global $U(1)$ symmetry is spontaneously broken and its current j_a has a pole in its correlation function. The unbroken gauged $U(1)$ has a massless pole corresponding to the photon. In addition, the two towers of intermediate states have the same masses with values (15). There is a family of more general mixed boundary conditions which are interesting and which will be examined in detail elsewhere.

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